Image Processing & Pattern

E1425

Lecture 6

Linear Image Restoration

INSTRUCTOR

DR / AYMAN SOLIMAN

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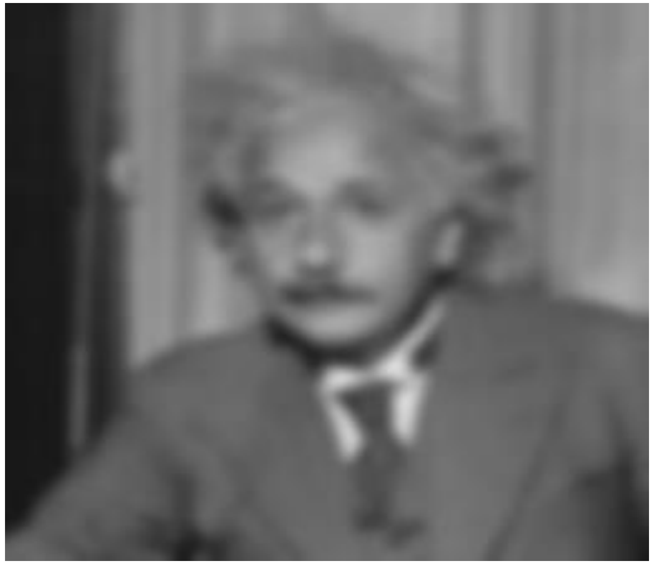
➢ Blur



out-of-focus blur

motion blur

Question 1: How do you know they are blurred?
I’ve not shown you the originals!

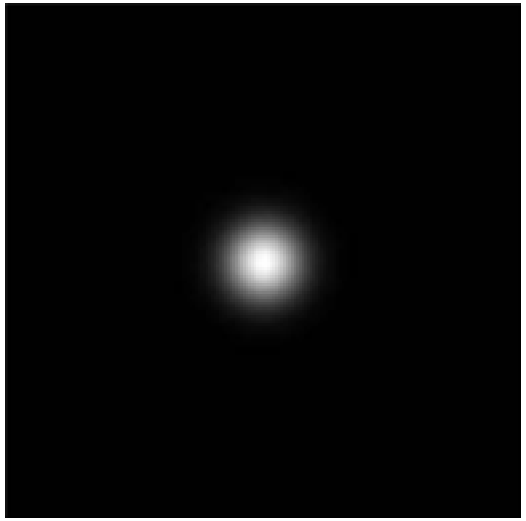
Question 2: How do I deblur an image?

➢ Linear Blur Model

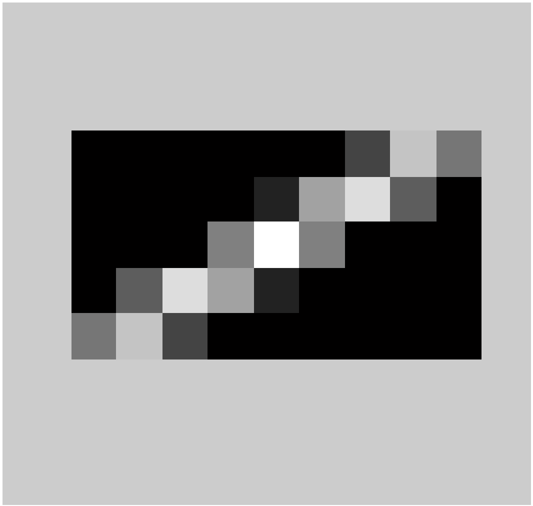
- Spatial domain

$$x(m,n) \longrightarrow \boxed{h(m,n)} \longrightarrow y(m,n)$$

blurring filter



Gaussian blur



motion blur

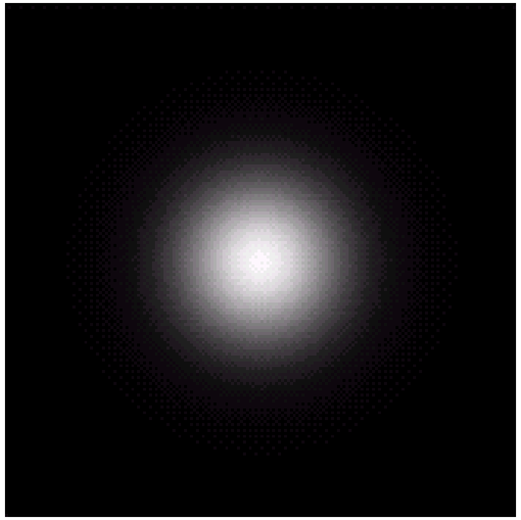
From Prof. Xin Li

➢ Linear Blur Model

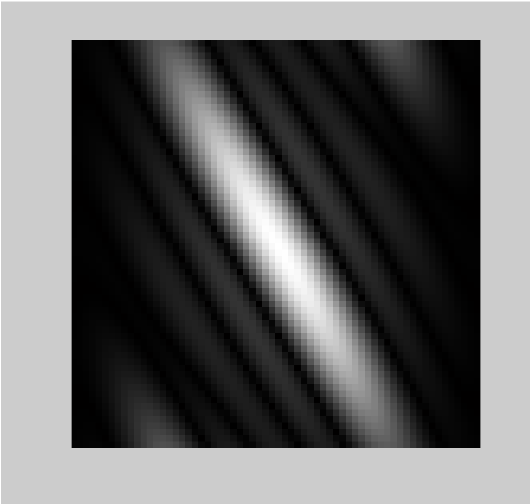
- Frequency (2D-DFT) domain

$$X(u,v) \longrightarrow \boxed{H(u,v)} \longrightarrow Y(u,v)$$

blurring filter



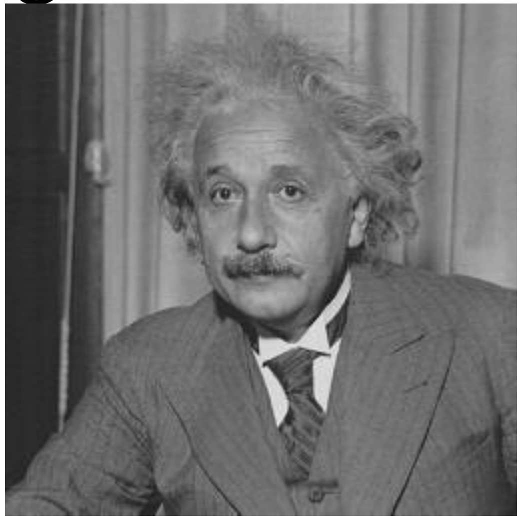
Gaussian blur



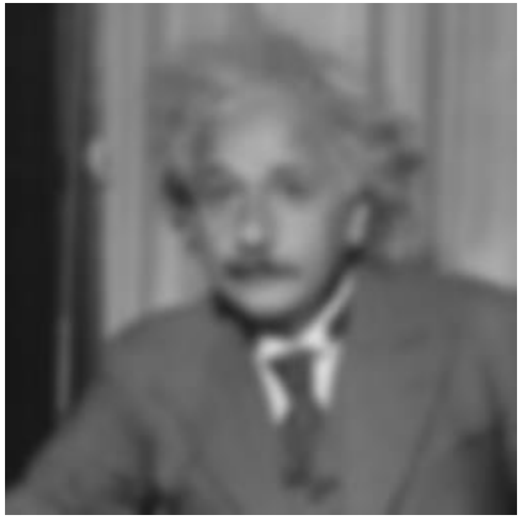
motion blur

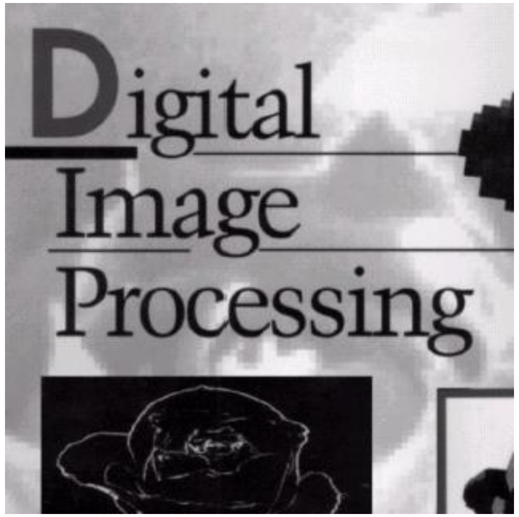
From Prof. Xin Li

➢ Blurring Effect

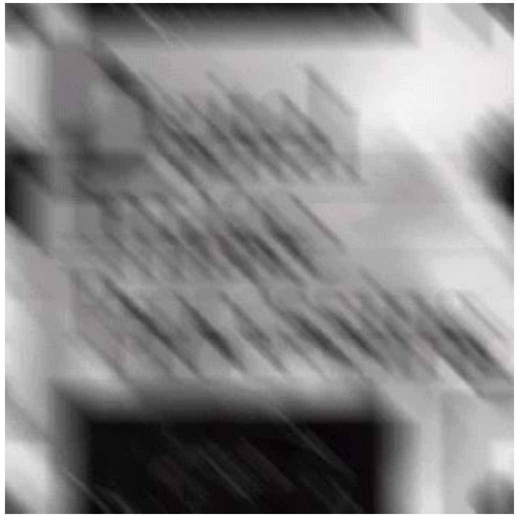


Gaussian blur →

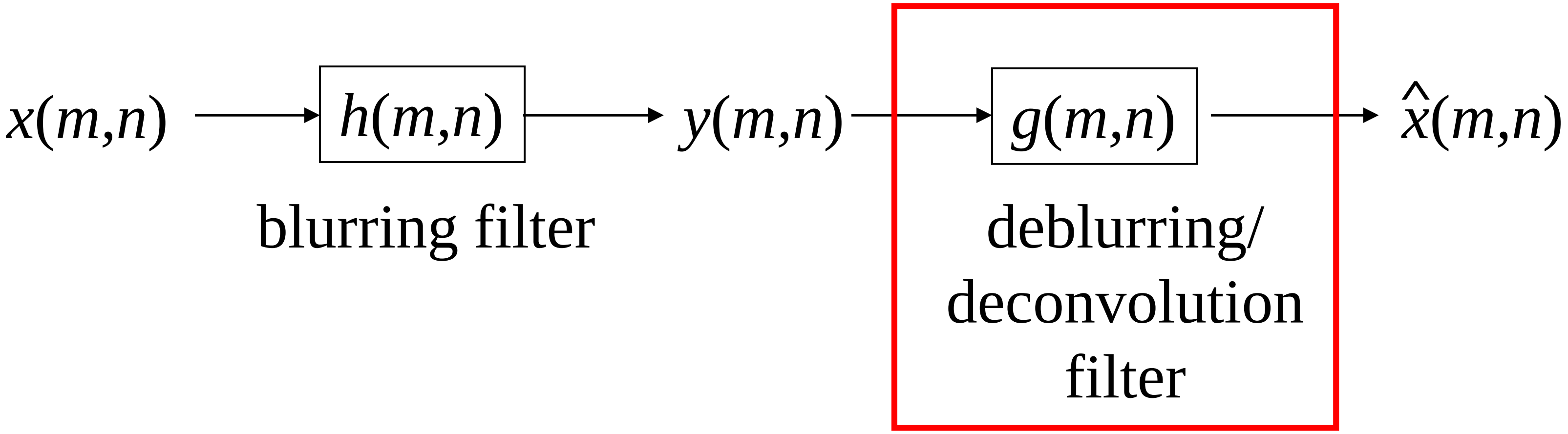




motion blur →



➢ Image Restoration: Deblurring/Deconvolution



- **Non-blind deblurring/deconvolution**

  Given: observation y(m,n) and blurring function h(m,n)

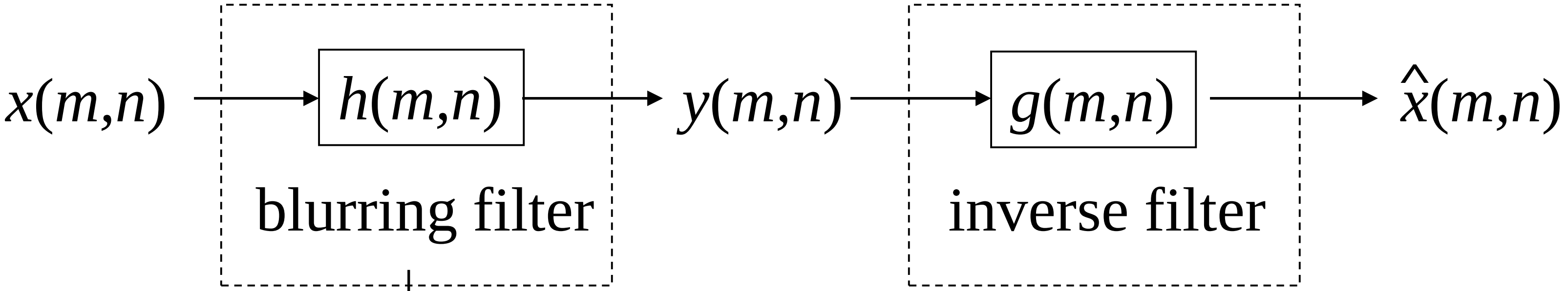
  Design: g(m,n), such that the distortion between x(m,n) and is minimized

- **Blind deblurring/deconvolution**

  Given: observation y(m,n)

  Design: g(m,n), such that the distortion between x(m,n) and is minimized

➢ Deblurring: Inverse Filtering



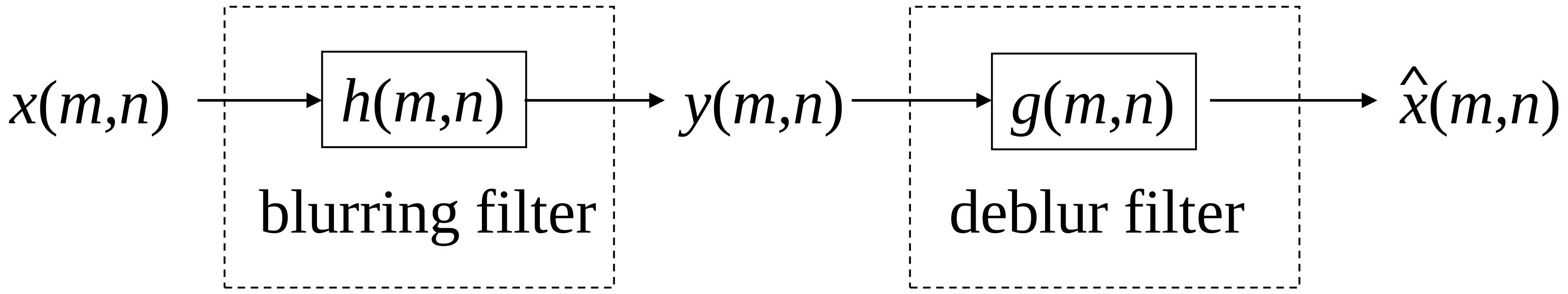
$$X(u,v)\,H(u,v) = Y(u,v)$$

$$X(u,v) = \frac{Y(u,v)}{H(u,v)} = \frac{1}{H(u,v)}\,Y(u,v)$$

$$G(u,v) = \frac{1}{H(u,v)}$$

Exact recovery!

➢ Deblurring: Pseudo-Inverse Filtering



Inverse filter: $G(u,v) = \dfrac{1}{H(u,v)}$

What if at some (u,v), $H(u,v)$ is 0 (or very close to 0) ?

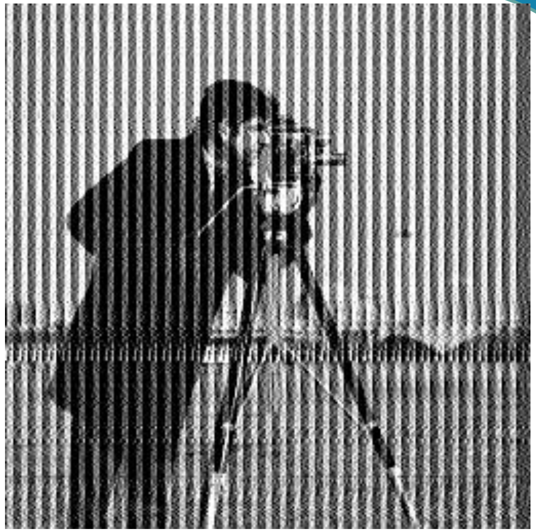
Pseudo-inverse filter:

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & |H(u,v)| > \delta \\ 0 & |H(u,v)| \le \delta \end{cases}$$

small threshold

➢ Inverse and Pseudo-Inverse Filtering

$$G(u,v) = \frac{1}{H(u,v)}$$

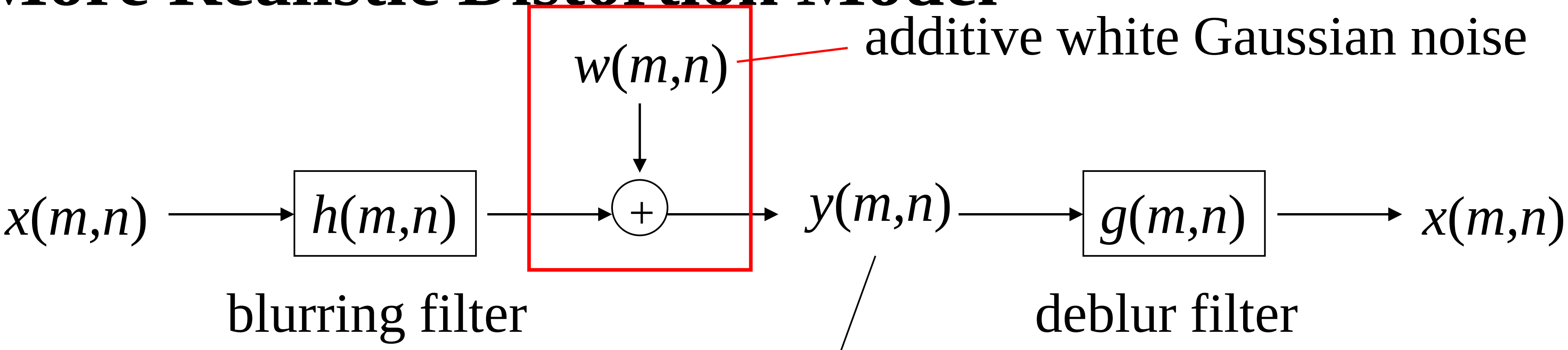


$$G(u,v) = \begin{cases} \frac{1}{H(u,v)} & |H(u,v)| > \delta \\ 0 & |H(u,v)| \le \delta \end{cases}$$

$\delta = 0.1$

Adapted from Prof. Xin Li

➢ More Realistic Distortion Model



$$Y(u,v) = X(u,v)\,H(u,v) + W(u,v)$$

- **What happens when an inverse filter is applied?**

$$\hat{X}(u,v) = Y(u,v)G(u,v) = \frac{X(u,v)H(u,v) + W(u,v)}{H(u,v)}$$

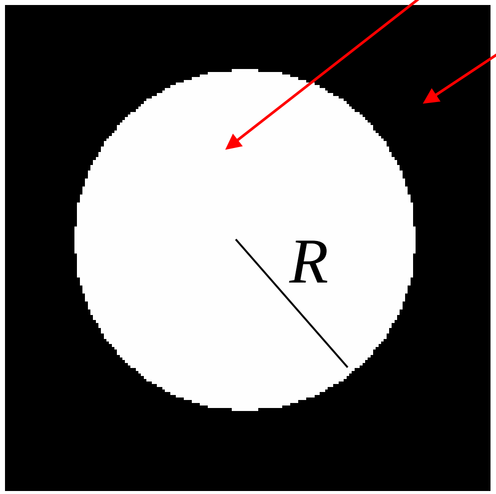
$$= X(u,v) + \frac{W(u,v)}{H(u,v)}$$

close to zero at high frequencies

➢ Radially Limited Inverse Filtering

Radially limited inverse filter:

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & \sqrt{u^2+v^2} \le R \\ 0 & \sqrt{u^2+v^2} > R \end{cases}$$



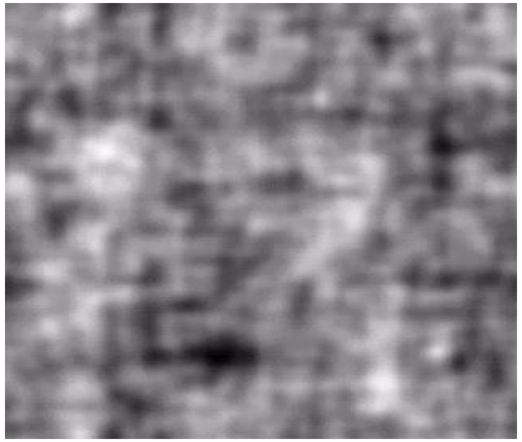
- **Motivation**
  - Energy of image signals is concentrated at low frequencies
  - Energy of noise uniformly is distributed over all frequencies
  - Inverse filtering of image signal dominated regions only

➢ Radially Limited Inverse Filtering

Image size:
480x480

Original

Blurred

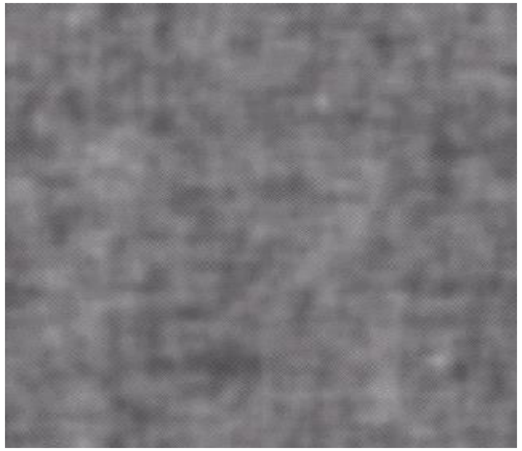


Inverse filtered

Radially limited inverse filtering:

$R = 40$

$R = 70$



$R = 85$

Wiener (Least Square) Filtering

Wiener filter:

$$G(u,v) = \frac{H*(u,v)}{|H(u,v)|^2 + K}$$

$$K = \frac{\sigma_W^2}{\sigma_X^2}$$

σ_W^2 → noise power

σ_X^2 → signal power

- Optimal in the least MSE sense, i.e.
  $G(u, v)$ is the best possible linear filter that minimizes

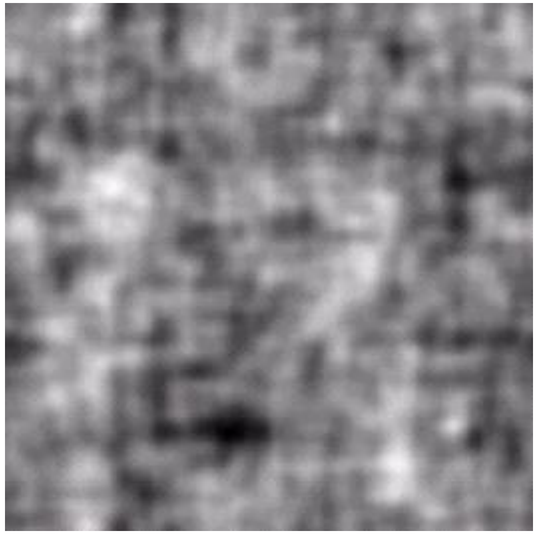
$$errorenergy = E\left\{\left|\hat{X}(u,v) - X(u,v)\right|^2\right\}$$

- Must estimate signal and noise power

➢ Weiner Filtering

Blurred image

Inverse filtering



Radially limited inverse filtering
$R = 70$

Weiner filtering

From [Gonzalez & Woods]

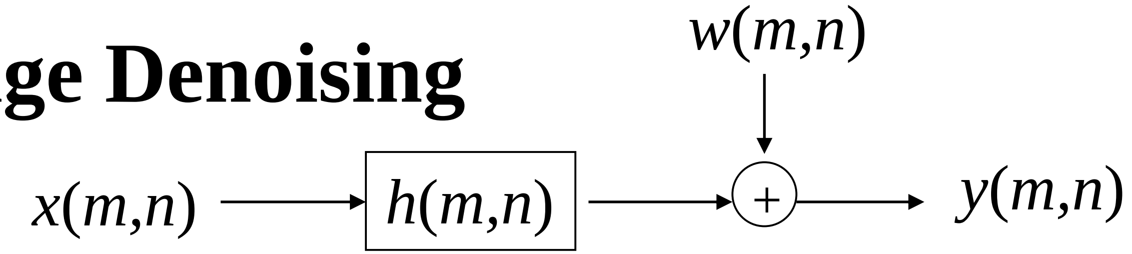
➢ Inverse vs. Weiner Filtering

distorted   inverse filtering   Wiener filtering

motion blur
+
noise
↓
less noise
↓
less noise

From [Gonzalez & Woods]

➢ Weiner Image Denoising



- What if no blur, but only noise, i.e., $h(m,n)$ is an impulse or $H(u, v) = 1$?

Wiener filter:

$$G(u,v) = \frac{H*(u,v)}{|H(u,v)|^2 + K}$$

where $K = \frac{\sigma_W^2}{\sigma_X^2}$

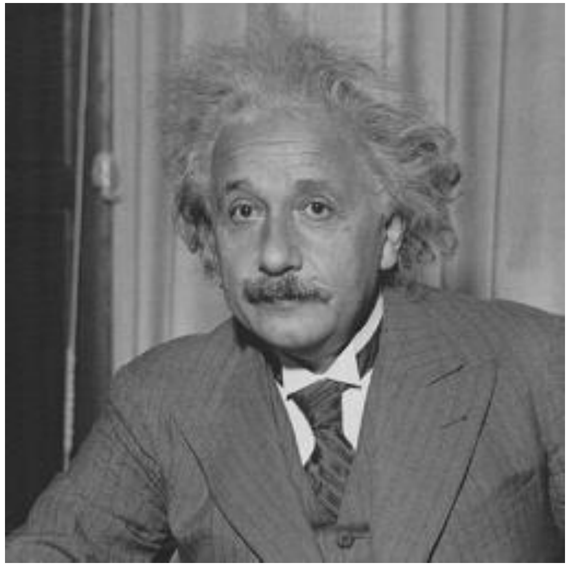
for $H(u,v) = 1$

Wiener denoising filter:

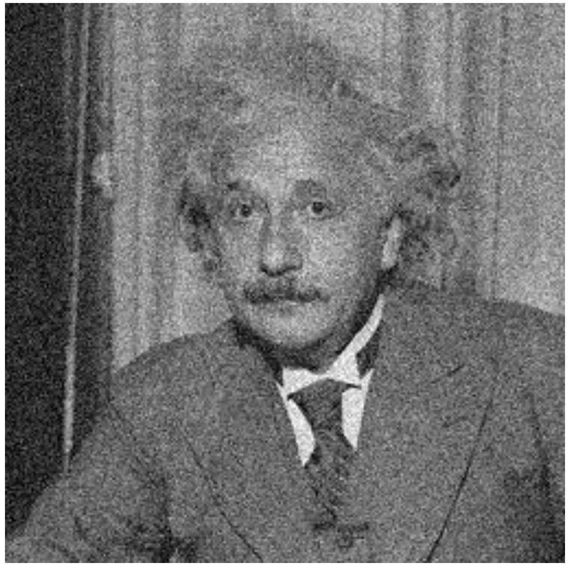
$$G(u,v) = \frac{1}{1+K} = \frac{1}{1+\sigma_W^2/\sigma_X^2} = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_W^2}$$

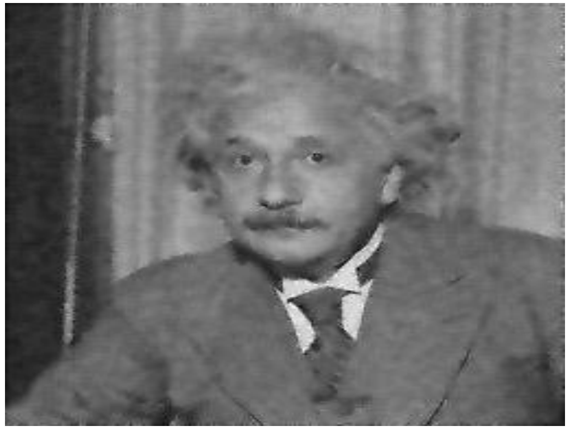
Typically applied locally in space

➢ Weiner Image Denoising



adding noise
noise var = 400





local Wiener denoising

➢ Summary of Linear Image Restoration Filters

Inverse filter:

$$G(u,v) = \frac{1}{H(u,v)}$$

Pseudo-inverse filter:

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & |H(u,v)| > \delta \\ 0 & |H(u,v)| \leq \delta \end{cases}$$

Radially limited inverse filter:

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & \sqrt{u^2+v^2} \leq R \\ 0 & \sqrt{u^2+v^2} > R \end{cases}$$

Wiener filter:

$$G(u,v) = \frac{H*(u,v)}{|H(u,v)|^2 + K}$$ where $K = \frac{\sigma_W^2}{\sigma_X^2}$

Wiener denoising filter:

$$G(u,v) = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_W^2}$$

➢ Examples

- A blur filter *h*(*m*,*n*) has a 2D-DFT given by

$$H(u,v) = \begin{bmatrix} 1 & -0.3-0.3j & 0 & -0.3+0.3j \\ -0.3-0.3j & 0.1j & 0 & 0.1 \\ 0 & 0 & 0 & 0 \\ -0.3+0.3j & 0.1 & 0 & -0.1j \end{bmatrix}$$

- Find the deblur filter *G*(*u*,*v*) using

1) The inverse filtering approach
2) The pseudo-inverse filtering approach, with $\delta = 0.05$
3) The pseudo-inverse filtering approach, with $\delta = 0.2$
4) Wiener filtering approach, with $\sigma_X^2 = 625$ and $\sigma_W^2 = 125$

Examples

1) Inverse filter

$$G(u,v) = \frac{1}{H(u,v)} = \begin{bmatrix} 1 & -1.67+1.67j & Inf & -1.67-1.67j \\ -1.67+1.67j & -10j & Inf & 10 \\ Inf & Inf & Inf & Inf \\ -1.67-1.67j & 10 & Inf & 10j \end{bmatrix}$$

2) Pseudo-inverse filter, with $\delta = 0.05$

$$G(u,v) = \begin{cases} \frac{1}{H(u,v)} & |H(u,v)| > \delta \\ 0 & |H(u,v)| \le \delta \end{cases} = \begin{bmatrix} 1 & -1.67+1.67j & 0 & -1.67-1.67j \\ -1.67+1.67j & -10j & 0 & 10 \\ 0 & 0 & 0 & 0 \\ -1.67-1.67j & 10 & 0 & 10j \end{bmatrix}$$

➢ Examples

3) Pseudo-inverse filter, with $\delta = 0.2$

$$G(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & |H(u,v)| > \delta \\ 0 & |H(u,v)| \le \delta \end{cases} = \begin{bmatrix} 1 & -1.67+1.67j & 0 & -1.67-1.67j \\ -1.67+1.67j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1.67-1.67j & 0 & 0 & 0 \end{bmatrix}$$

4) Wiener filter, with $\sigma_X^2 = 625$ and $\sigma_W^2 = 125$

$$K = \frac{\sigma_W^2}{\sigma_X^2} = \frac{125}{625} = 0.2$$

$$G(u,v) = \frac{H*(u,v)}{|H(u,v)|^2 + K} = \begin{bmatrix} 0.83 & -0.79+0.79j & 0 & -0.79-0.79j \\ -0.79+0.79j & -0.48j & 0 & 0.48 \\ 0 & 0 & 0 & 0 \\ -0.79-0.79j & 0.48 & 0 & 0.48j \end{bmatrix}$$

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